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Ques : State and prove Carnot's theorem. (2011, 2013, 2015)

Ans : Statement of Carnot's theorem : From second law of thermodynamics , two conclusions are obtained which are used to constitute Carnot's theorm.

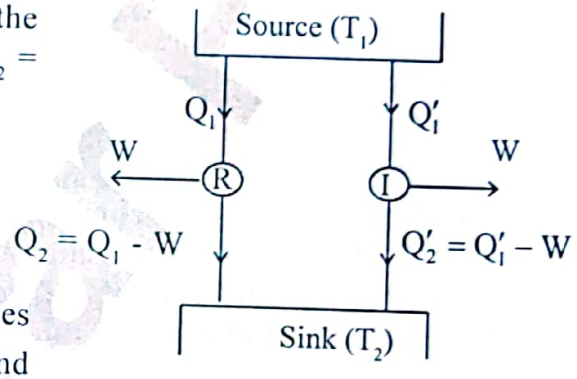
1. No engine will be more efficient than a perfectly reversible engine working between the same two temperatures (between the same source and the sink)

2. The efficiency of all reversible engines working between the same two temperatures (the same source and the sink) is same whatever the working substance.

Proof :

First part : For proving first part , we consider two heat engines R (reversible engine) and I (Irreversible engine) working between the same source (T_1) and the same sink (T_2) where $T_1 > T_2$.

Suppose reversible engine R takes heat Q_1 from the source (T_1), performs work W and rejects heat $Q_2 = Q_1 - W$ to the sink (T_2).



Efficiency of the reversible engine R is

$$\eta_R = \frac{W}{Q_1}$$

Similarly suppose the Irreversible engine I takes heat Q_1' from the source (T_1), performs work W and rejects heat $Q_2' = Q_1' - W$ to the sink (T_2).

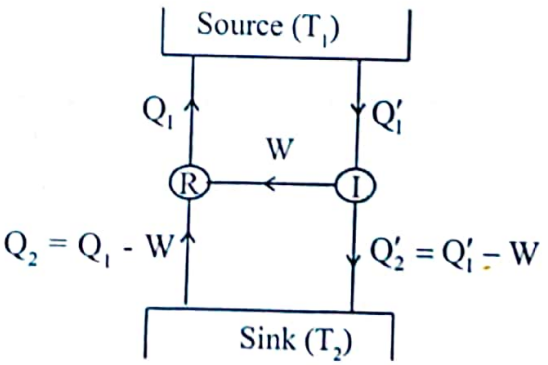
Efficiency of the Irreversible engine I is

$$\eta_I = \frac{W}{Q_1'}$$

Suppose the Irreversible engine I is more efficient than the reversible engine R

$$\text{i.e, } \eta_I > \eta_R \Rightarrow \frac{W}{Q_1'} > \frac{W}{Q_1} \Rightarrow Q_1 > Q_1' \Rightarrow Q_1 - Q_1' = \text{a positive quantity} \quad \ldots \quad (1)$$

Suppose the two engines are coupled together so that the engine R works in opposite direction i.e, the engine R works as a refrigerator. The engine I absorbs heat Q_1' from the source (T_1), performs work W and rejects heat $Q_2' = Q_1' - W$ to the sink (T_2). The engine R takes heat Q_2 from the sink (T_2) , W work is done on the engine R and $Q_1 = Q_2 + W$ heat is rejected to the source (T_1). Here we can say that work done on the gas by the engine R is actually received from the work done by the gas on the engine I. Thus both the engines R and I work as a self acting machine.



Heat lost by the sink is $Q_2 - Q_2' = (Q_1 - W) - (Q_1' - W) \Rightarrow Q_1 - Q_1' = \text{a positive quantity}$

i.e, External work done on the system = 0

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Thus the coupled engines forming a self acting machine unaided by any external agency transfers heat continuously from a body at lower temperature to the body at higher temperature. This conclusion is contrary to the second law of thermodynamics. i.e. our assumption $\eta_I > \eta_R$ is wrong. Therefore no engine will be more efficient than a perfectly reversible engineworking between the same two temperatures (the same source and The same sink).

Second part : We consider two reversible engines R_1 and R_2 working between the same source and the same sink (i.e. between the same two temperatures). Suppose R_1 drives R_2 backward then R_1 can not be more efficient than R_2 and suppose if R_2 drives R_1 backward then R_2 can not be more efficient than R_1. Hence the two reversible engines R_1 and R_2 are equally efficient. Thus all reversible engines working between the same two temperatures will have same efficiency. Proved